Quiz 9b Calculus Score: Answer Key Name 2, 3, 4, 6, 8

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the Maclaurin series for the given function.

NC 1) e^{3x} 1) A

A) $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$ B) $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n x^n}{n!}$

C) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!}$ D) $\sum_{n=1}^{\infty} \frac{3^n x^n}{n!}$

Objective: (9.3) Find Maclaurin Series

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$, $e^{3x} = 1 + 3x + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \dots + \frac{(3x)^n}{n!} + \dots$

NC ~~2)~~ $\cos(-2x)$ 2) A

A) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}$ B) $-\sum_{n=0}^{\infty} \frac{(-1)^{2n} 2^{2n} x^{2n}}{(2n)!}$

C) $\sum_{n=0}^{\infty} \frac{(-1)^{2n} 2^{2n} x^{2n}}{(2n)!}$ D) $-\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n}}{(2n)!}$

Objective: (9.3) Find Maclaurin Series

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots$

$\cos(-2x) = 1 - \frac{(-2x)^2}{2!} + \frac{(-2x)^4}{4!} - \frac{(-2x)^6}{6!} + \dots + (-1)^n \frac{(-2x)^{2n}}{(2n)!} + \dots \rightarrow \frac{(-1)^n(-2)^{2n}(x^{2n})}{(2n)!} = \frac{(-1)^n(2)^{2n}(x^{2n})}{(2n)!}$

Find the Taylor polynomial of the indicated order and use it to approximate the function with the given value.

C ~~3)~~ sin 0.195 (3) 3) C

A) 0.1760477 B) 0.1925848 C) 0.1937665 D) 0.1962382

Objective: (9.3) Use Taylor Polynomial to Approximate Function

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$, $\sin(0.195) \approx 0.195 - \frac{0.195^3}{3!} = 0.1937641875$

Solve the problem.

~~4)~~ Calculate the magnitude of the approximation error of $e^{0.3}$ using four terms of the Maclaurin series expansion. 4) ________

A) 6.8×10^{-5} B) 2.0×10^{-5} C) 1.0×10^{-4} D) 3.6×10^{-4}

Objective: (9.3) Estimate Error of Truncated Taylor Series

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

$\text{error} = e^{.3} - \left(1 + .3 + \frac{.3^2}{2!} + \frac{.3^3}{3!}\right)$

$= 3.5888075 \times 10^{-4}$

$\boxed{\text{Calculator}}$

$R_3(x) = \frac{f^4(c)}{4!} x^4$

$R_3(.3) = \frac{e^{.3}}{4!}(.3)^4$

$= 4.555773 \times 10^{-4}$

$\boxed{\text{Taylor Remainder Theorem}}$

$y = e^x$
$y' = e^x$
$y^2 = e^x$
$y^3 = e^x$
$y^4 = e^x$

5) The polynomial $1 + 7x + 21x^2$ is used to approximate $f(x) = (1 + x)^7$ on the interval $-0.01 \le x \le 0.01$. Use the Remainder Estimation Theorem to estimate the maximum absolute error. 5) C

A) $\approx 3.642 \times 10^{-4}$ B) $\approx 2.061 \times 10^{-5}$ C) $\approx 3.642 \times 10^{-5}$

Objective: (9.3) Estimate Error of Truncated Taylor Series

$f'(x) = 7(1+x)^6$

$f''(x) = 42(1+x)^5$

$f'''(x) = 210(1+x)^4$

$R_2(x) = \frac{|f'''(c)|}{3!}(x)^3$

$R_2(.01) = \frac{210(1+c)^4}{3!}(.01)^3$

$R_2(.01) = \frac{210(1+.01)^4(.01)^3}{3!} = 0.0000364211$

Determine convergence or divergence of the series.

6) $\sum_{n=0}^{\infty} \frac{n^3+8}{6^n} = 8 + \frac{9}{6} + \frac{16}{36} + \frac{35}{216} + \dots = 8 + 1.5 + 0.44 + 0.162 + \dots$ 6) A

A) Converges B) Diverges $-1 < \frac{1}{6} < 1$, converges

Objective: (9.4) Determine Convergence or Divergence

$\lim_{n\to\infty} \frac{(n+1)^3+8}{6^{\cancel{n+1}}} \cdot \frac{\cancel{6^n}}{n^3+8} = \lim_{n\to\infty} \frac{(n+1)^3+8}{6(n^3+8)} = \lim_{n\to\infty} \frac{3(n+1)^2}{18n^2} = \lim_{n\to\infty} \frac{6(n+1)}{36n} = \lim_{n\to\infty} \frac{6}{36} = \frac{1}{6}$

7) $\sum_{n=1}^{\infty} n!\, e^{-8n} = \frac{1!}{e^8} + \frac{2!}{e^{-16}} + \frac{3!}{e^{-24}} + \frac{4!}{e^{-32}} + \dots$ 7) A

A) Diverges B) Converges

Objective: (9.4) Determine Convergence or Divergence

$\lim_{n\to\infty} \frac{(n+1)\cancel{n!}}{\cancel{e^{8n+8}}} \cdot \frac{\cancel{e^{8n}}}{\cancel{n!}} = \lim_{n\to\infty} \frac{n+1}{e^8} = \infty$ diverges

Find the interval of convergence of the series.

$\lim_{n\to\infty} \frac{(x-2)^{2n+2}}{\cancel{4^{n+1}}} \cdot \frac{\cancel{4^n}}{\cancel{(x-2)^{2n}}} = \lim_{n\to\infty} \frac{(x-2)^2}{4}$

8) $\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{4^n}$ 8) A

A) $0 < x < 4$ B) $1 < x < 3$ C) $-4 < x < 4$ D) $x < 4$

Objective: (9.4) Find Interval of Convergence

$-1 < \frac{(x-2)^2}{4} < 1$, $\cancel{-4} < (x-2)^2 < 4$

$-2 < x-2 < 2$

$0 < x < 4$

9) $\sum_{n=0}^{\infty} \left(\frac{\sqrt{x}}{2} - 1\right)^n = 1 + \left(\frac{\sqrt{x}}{2} - 1\right)^1 + \left(\frac{\sqrt{x}}{2} - 1\right)^2 + \left(\frac{\sqrt{x}}{2} - 1\right)^3 + \dots$ 9) ______

A) $-2 < x < 2$ B) $0 < x < 4$ C) $0 < x < 2$ D) $-4 < x < 4$

Objective: (9.4) Find Interval of Convergence

$\lim_{n\to\infty} \frac{\left(\frac{\sqrt{x}}{2} - 1\right)^{n+1}}{\cancel{\left(\frac{\sqrt{x}}{2} - 1\right)^n}} = \lim_{n\to\infty} \frac{\sqrt{x}}{2} - 1 = \frac{\sqrt{x}}{2} - 1$

$-1 < \frac{\sqrt{x}}{2} - 1 < 1$

$0 < \frac{\sqrt{x}}{2} < 2$

$0 < \sqrt{x} < 4$, $0 < x < 16$