Discrete Random Variables, Sample Spaces, Probability Distribution, Means, and Variances

A discrete random variable is a countable outcome with determined probability. Lots of things in our world can be classified as discrete random variables. For example, rolling a die has six possible outcomes. Since we can count all the possible outcomes and there is a determined probability for each outcome, these are discrete random variables.

The list of all possible outcomes is called the sample space. A listing of the probabilities associated with each possible outcome is the probability distribution.

Let's create a sample space and probability distribution for this situation of rolling a die.

Sample Space	Probability Distribution
1	
2	
3	
4	
5	
6	

List some other examples of discrete random variables: ____________________

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In some situations we attach a weighted value to each possible outcome in the sample space.

Example 1 For instance, suppose we decide to award 2 points if you roll an even number on a die and subtract 1 point if you roll an odd number. Since the weighted values are different for each outcome in the sample space we may wonder what is the expected value or score for repeating this experiment many times.

a. What's your prediction for the expected value of this game if it is repeated many times?

a. less than 0
b. equal to 0
c. bigger than 0

Explain why you think this result will occur?

b. Let's try the experiment. Roll a die 60 times recording the outcomes and values based on the weighting values above.

Roll	Score	Roll	Score	Roll	Score	Roll	Score
1		16		31		46	
2		17		32		47	
3		18		33		48	
4		19		34		49	
5		20		35		50	
6		21		36		51	
7		22		37		52	
8		23		38		53	
9		24		39		54	
10		25		40		55	
11		26		41		56	
12		27		42		57	
13		28		43		58	
14		29		44		59	
15		30		45		60	

Find the average or mean score for all 60 rolls. ______________________________

We can predict an expected value for this situation. The expected value is the sum of the product of the value for each item in the sample space times its individual probability. We also call this the mean of the discrete random variable and it represents a weighted average of the possible values a random variable can take.

Finish the calculation started below to compute the expected value for this situation.

$$\text{Mean } (\mu_x) = (-1)\left(\frac{1}{6}\right) + (2)\left(\frac{1}{6}\right) + (-1)\left(\frac{1}{6}\right) + \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_} =$$

Compare this expected value with your actual score in rolling a die 60 times above. ________________

Once we have computed the expected value or mean of a random variable situation, we often want to know how far away from this expected value we are in an actual set of data. We can calculate a measure of how spread out from the expected value our data should be. We call this the variance of a discrete random variable. This means we are predicting the difference between an actual mean score and the predicted expected value. Recall that the expected value is the value we expect when repeating an event over a large number of trials. This variance is computed using the following formula where x_i is the value of an individual outcome in the sample space, μ_x is the expected value, and p_i is the predicted probability of the outcome.

$$\text{Variance } \sigma^2 = \Sigma\left(x_i - \mu_x\right)^2\left(p_i\right)$$
$$= \left(x_1 - \mu_x\right)^2\left(p_1\right) + \left(x_2 - \mu_x\right)^2\left(p_2\right) + \left(x_3 - \mu_x\right)^2\left(p_3\right) + \ldots$$

Find the variance for the situation described in example 1. The calculation is started below; you finish it.

$$\text{Variance } \sigma^2 = (\text{roll a 1 score: -1} - \text{mean: } 0.5)^2 (p_1) + (\text{roll a 2 score: } 2 - \text{mean: } 0.5)^2 (p_2) + (\text{roll a 3 score: -1} - \text{mean: } 0.5)^2 (p_3) + \ldots$$

$$= (-1 - 0.5)^2 \left(\frac{1}{6}\right) + (2 - 0.5)^2 \left(\frac{1}{6}\right) + \ldots$$

The value of the variance simply tells us about how spread out our actual mean score may be. The larger the value of the variance, the greater the spread in the mean score. The standard deviation is the square root of the variance.

<u>Example 2</u> Consider rolling two dice and finding the sum of both dice. We award -2 points for a sum less than or equal to 4 and +3 points for a sum greater than 4.

a. Find the sample space for this situation.
b. Find the probability distribution for this situation.
c. Calculate the mean or expected value for this situation.
d. Find the variance of distribution for this situation.

Helmets

Suppose you own a business that makes and sells football helmets. Every perfectly made helmet is sold for $50. Helmets with small blemishes are discounted to $30. For all defective helmets that cannot be sold, you loose $20. Assume the probability of getting a defective helmet that cannot be sold is 1 out of 10, the probability of getting a helmet with a small blemish is 1 out of 5, and perfect helmets is 7 out of 10.

a) Create a table/chart showing the sample space and probability of distribution.

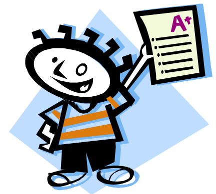
b) Find the expected income (mean income) for manufacturing this helmet over an extended period of time.

c) Find the variance of distribution for this situation. What does the variance tell us about this situation? Explain.

d) In one month you made 150 helmets. You sold 128 helmets. Of the ones you sold, only 101 were perfect. Find the actual mean earnings for making the 150 helmets.

e) Summary of class discussion:

How Much Is My Report Card Worth?



A student is paid \$4.00 for each A they earn, \$3.00 for each B, \$2.00 for each C, but the student must pay their parents \$5.00 for each F. Suppose the probability of earning an A in Mr. Jones' Precalculus class is 20%, the probability of a B is 30%, the probability of a C is 45%, and the probability of earning an F is 5%.

a) Obtain and organize into a table the sample space and probability distribution.

b) Compute the expected (mean) to learn how much a random student can expect to earn for their grades in their Precalculus class.

c) Compute the variance. What does the variance tell us about this situation? Explain.

d) At the end of term 4, Mr. Jones had given 6 A's, 8 B's, 10 C's, and 6 F's. Find the average earnings for these 30 students.

e) How does this mean compare to the expected value?